Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 5

with Solution

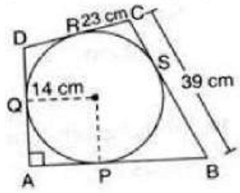
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, quadrilateral ABCD is circumscribed, touching the circle at P, Q, R and S such that $\angle DAB = 90^o$, If CR = 23 cm and CB = 39 cm and the radius of the circle is 14 cm, then the measure of AB is **[1]**



a) 37 cm b) 16 cm

c) 30 cm d) 39 cm

2. If the coordinates of a point are (3, -7), then its ordinate is **[1]**

a) 7 b) -3

c) -7 d) 3

3. The distance of the point (4, 7) from the x-axis is **[1]**

a) 7 b) 4

c) 11 d) $\sqrt{6}5$

4. Raju bought a fish from a shop for his aquarium. The shop keeper takes out one fish from a tank containing 15 male fish and 18 female fish. The probability that the fish taken out is a male fish is **[1]**

a) $\frac{5}{11}$ b) $\frac{6}{11}$

c) $\frac{5}{12}$

d) $\frac{7}{11}$

5. The point where the medians of a triangle meet is called the ________ of the triangle **[1]**

a) circumcentre

b) centroid

c) orthocentre

d) None of these

6. 4 chairs and 3 tables cost Rs.2100 and 5 chairs and 2 tables costs Rs1750. The cost of a chair is **[1]**

a) Rs.500

b) Rs.350

c) Rs.250

d) Rs.150

7. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is **[1]**

a) 2.1cm

b) 4.2cm

c) 1.05cm

d) 8.4cm

8. Someone is asked to take a number from 1 to 100. The probability that it is a prime is **[1]**

a) $\frac{1}{40}$

b) $\frac{1}{5}$

c) $\frac{1}{4}$

d) $\frac{6}{25}$

9. Which one of the following is not a quadratic equation? **[1]**

a) $x^2 + 3x = (-1)(1 - 3x)^2$

b) $(x + 2)^2 = 2(x + 3)$

c) $(x + 2)(x - 1)=x^2 - 2x - 3$

d) $x^3 - x^2 + 2x + 1 = (x + 1)^3$

10. If P(E) = 0.05, what will be the probability of 'not E'? **[1]**

a) 0.55

b) 0.59

c) 0.95

d) 0.095

11. $\frac{\sec 30^\circ}{cosec\ 60^\circ} = ?$ **[1]**

a) $\frac{2}{\sqrt{3}}$

b) 1

c) $\frac{\sqrt{3}}{2}$

d) $\sqrt{3}$

12. HCF of $(2^3 \times 3^2 \times 5)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3 \times 5^3 \times 7)$ is **[1]**

a) 60

b) 48

c) 30

d) 105

13. The values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is **[1]**

a) 0 only
b) 8 only
c) 0, 8
d) 4

14. A circus artist is climbing a 30 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. If the angle made by the rope with the ground level is 45^o, then the height of the pole is **[1]**

a) $20\sqrt{2}$ m
b) $15\sqrt{2}$ m
c) $10\sqrt{2}$ m
d) 20 m

15. The distance between the points (3, -2) and (-3, 2) is: **[1]**

a) 40
b) $4\sqrt{10}$
c) $2\sqrt{10}$
d) $\sqrt{52}$

16. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is **[1]**

a) 205400
b) 203400
c) 194400
d) 198400

17. The relation between mean, mode and median is **[1]**

a) mode = (3 × mean) - (2 × median)
b) mode = (3 × median) - (2 × mean)
c) mean = (3 × median) - (2 × mode)
d) median = (3 × mean) - (2 × mode)

18. If a pair of linear equations is inconsistent then their graph lines will be **[1]**

a) always coincident
b) parallel
c) always intersecting
d) intersecting or coincident

19. **Assertion (A):** In △ABC, DE || BC such that AD = (7x - 4) cm, AE = (5x - 2) cm, DB = (3x + 4) cm
and EC = 3x cm than x equal to 5. **[1]**
Reason (B): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distant point, than the other two sides are divided in the same ratio.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** For any two positive integers a and b, HCF(a, b) × LCM(a, b) = a × **[1]**

b

Reason (R): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve the following pair of linear equations by the substitution method: $x + y = 14$, $x - y = 4$ **[2]**

22. Find a quadratic polynomial, the sum and product of whose zeroes are $0, \sqrt{5}$ respectively. **[2]**

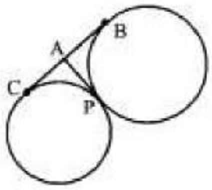
23. A group consists of 12 persons, out of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is **[2]**

i. extremely patient

ii. extremely kind or honest.

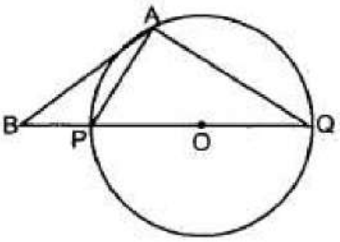
Which of the above you prefer more?

24. In the adjoining figure, BC is a common tangent to the given circles which touch externally at P. Tangent at P meets BC at A. If BA = 2.8 cm, then what is the length of BC? **[2]**



OR

The tangent at a point A of a circle with centre O intersects the diameter PQ of the circle(when extended) at the point B. If $\angle BAQ = 105^{o}$, find APQ.



25. What is the distance between the points (5 sin 60°, 0) and (0, 5 sin 30°)? **[2]**

OR

Find a point on the y-axis which is equidistant from the points A(6, 5) and B (- 4, 3).

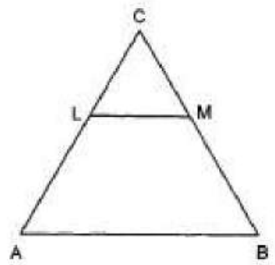
Section C

[3]

26. If 5 tan a = 4, show that $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta} = \frac{1}{6}$

27. Solve the system of equations by using the method of substitution: [3]
x + 2y = -1
2x - 3y = 12

28. In Fig. $LM \parallel AB$. If AL = x - 3, AC = 2x, BM = x - 2 and BC = 2x + 3, find the value of x. [3]



29. Prove that $\sqrt{2}$ is an irrational number. [3]

OR

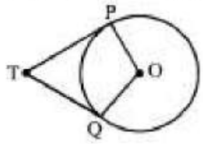
If two positive integers p and q are written as p = a^2b^3 and q = a^3 b, a and b are a prime number then. Verify.LCM $\times$(p.q.) $\times$ HCF (p.q.) = pq

30. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}m$, find the speed in km/hr of the plane. [3]

31. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus. [3]

OR

In the adjoining figure, TP and TQ are tangents to the circle with centre O such that $\angle POQ = 110^\circ$. Then find $\angle PTQ$.



Section D

32. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio. [5]

33. At t minutes past 2 p.m, the time needed by the minute hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t. [5]

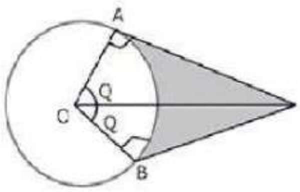
OR

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

34. Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130π cm^2. Find the radius each bead. **[5]**

OR

An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area.



35. Calculate the median from the following frequency distribution: **[5]**

Class	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45
Frequency	5	6	15	10	5	4	2	2

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 30th instalment?

(ii) If he increases the instalment by 100 rupees every month, then what amount of loan does he still have to pay after 30th instalment?

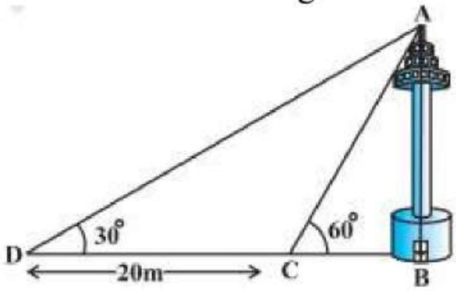
OR

If he increases the instalment by 200 rupees every month, then what amount would he pay in 40th instalment?

(iii) If he increases the instalment by 100 rupees every month, then what amount will be paid by him in the 100th instalment?

37. **Read the text carefully and answer the questions:** **[4]**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30°.



(i) Find the width of the canal.

(ii) Find the height of tower.

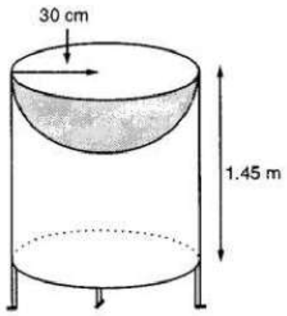
OR

Find the distance between top of tower and point C.

(iii) Find the distance between top of the tower and point D.

38. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

(iii) What is total cost for making the bird bath?

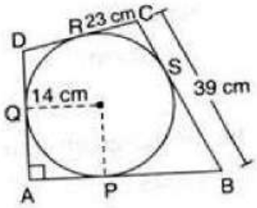
OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

Solution

Section A

1. **(c)** 30 cm

   Explanation: 

   Let the centre of the circle be O.
   Construction: Joined OP.
   ∵ Tangent is perpendicular to the radius through the point of contact.
   ∴ $\angle OQA = \angle OPA = 90^\circ$ and OQ = OP [Radii]
   ∴ OQAP is a square.
   ∴ AP = 14 cm
   Now, CR = CS = 23 cm [Tangents from an external point to a circle are equal]
   ∴ BS = 39 - 23 = 16 cm
   And BS = BP = 16 cm [Tangents from an external point to a circle are equal]
   Now, AB = AP + BP = 14 + 16 = 30 cm
2. **(c)** -7
   Explanation: Since y-coordinate of a point is called ordinate. Its distance from the x-axis measured parallel to the y-axis
   Therefore, the ordinate is -7.
3. **(a)** 7
   Explanation: The distance of the point (4, 7) from x-axis = 7
4. **(a)** $\frac{5}{11}$
   Explanation: Total number of fish = 15 + 18 = 33
   Male fish = 15
   Number of possible outcomes = 15
   Number of total outcomes = 15 + 18 = 33
   Required Probability = $\frac{15}{33} = \frac{5}{11}$
5. **(b)** centroid
   Explanation: The point where three medians of a triangle meet is called the centroid of the triangle. It is the centre of gravity of the triangle. It divides the median in the ratio 2 :1
6. **(d)** Rs.150
   Explanation: Let the cost of one chair be Rs. x x and the cost of 1 table be Rs. y. y.
   According to question,
   $4x + 3y = 2100$ ...(i)
   $5x + 2y = 1750$ ...(ii)
   Solving by elimination method,

$$\begin{array}{r} 8x + 6y = 4200 \\ 15x + 6y = 5250 \\ - \quad - \qquad - \\ \hline -7x = -1050 \\ x = 150 \end{array}$$

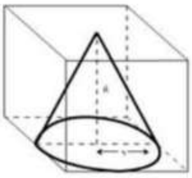
   Hence, the cost of a chair is Rs. 150

7. **(a)** 2.1cm

Explanation: Given: edge of the cube = 4.2 cm

A right circular cone is a Cone whose height is perpendicular to the diameter (radius) of the base circle.

In a cube, a largest right circular Cone is formed when its base lies on one of the faces of the Cube and its tip lies on the opposite face.



$\therefore$ Diameter of largest right circular Cone in Cube = edge length of cube.

$\therefore$ Diameter = 4.2 cm

$\Rightarrow$ Radius = $\frac{\text{diameter}}{2} = \frac{4.2}{2} = 2.1$ cm

$\therefore$ Radius of the largest right circular Cone in Cube is 2.1 cm

8. **(c)** $\frac{1}{4}$

Explanation: Total numbers of outcomes = 100

So, the prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

$\therefore$ Total number of possible outcomes = 25

$\therefore$ Required probability = 25/100 =$\frac{1}{4}$

9. **(c)** $(x + 2)(x - 1)=x^2 - 2x - 3$

Explanation: Degree of the equation is more than 2 i.e. 3.

10. **(c)** 0.95

Explanation: We know that

P(E) + P(not E) = 1

$\therefore$ P(not E) = 1 - P(E)

= 1 - 0.05

= 0.95

11. **(b)** 1

Explanation: We have, $\frac{\sec 30^\circ}{\csc 60^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 1$

12. **(a)** 60

Explanation: HCF = $(2^3 \times 3^2 \times 5, 2^2 \times 3^3 \times 5^2, 2^4 \times 3 \times 5^3 \times 7)$

HCF = Product of smallest power of each common prime factor in the numbers

$= 2^2 \times 3 \times 5 = 60$

13. **(c)** 0, 8

Explanation: If a quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has two equal roots, then its discriminant value will be equal to zero i.e., $D = b^2 - 4ac = 0$

Given, $2x^2 - kx + k = 0$

For equal roots,

$D = b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4(2)(k) = 0$

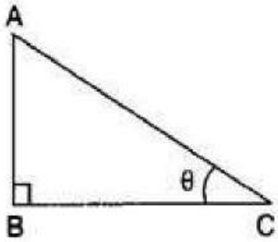
$\Rightarrow k^2 - 8k = 0$

$\Rightarrow k(k - 8) = 0$

$\therefore k = 0,8$

14. **(b)** $15\sqrt{2}$ m

Explanation:



Let AB be the height of the pole be h meters and length of rope = AC = 30 m

An angle of elevation = $\theta = 45^\circ$

$\therefore \sin 45^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{h}{30}$

$\Rightarrow h = \frac{30}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

= $15\sqrt{2}$ m

15. **(d)** $\sqrt{52}$

Explanation: Let us take (3, – 2) and (– 3, 2) as (x_1, y_1) and (x_2, y_2)

Using distance formjula, d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d = $\sqrt{(-3-3)^2 + (2-(-2)\ )^2}$

d = $\sqrt{(-6)^2 + (2+2)^2}$

d = $\sqrt{36 + (4)^2}$

d = $\sqrt{36 + 16}$

d = $\sqrt{52}$

16. **(c)** 194400

Explanation: Let the HCF of the numbers be x and their LCM be y.

It is given that the sum of the HCF and LCM is 1260, therefore

x + y = 1260 ....(i)

And, LCM is 900 more than HCF.

y = x + 900 ..... (ii)

Substituting (ii) in (i), we get:

x + x + 900 =1260

⇒ 2x + 900 = 1260

⇒ 2x = 1260 - 900

⇒2x = 360

⇒ x = 180

Substituting x = 180 in (i), we get:

y = 180 + 900

⇒ y = 1080

We also know that the product the two numbers is equal to the product of their LCM and HCF

Thus, product of the numbers = 1080(180) = 194400

17. **(b)** mode = (3 × median) - (2 × mean)

Explanation: mode = (3 × median) - (2 × mean)

18. **(b)** parallel

Explanation: We know that, If a pair of linear equations is inconsistent then their graph lines do not intersect each other and there will be no solution exists. Hence, the lines are parallel.

19. **(d)** A is false but R is true.

Explanation: We have,

$\frac{AD}{DB} = \frac{AE}{EC}$
$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$
$21x^2 - 12x = 15x^2 + 20x - 6x - 8$
$6x^2 - 26x + 8 = 0$
$3x^2 - 13x + 4 = 0$
$3x^2 - 12x - x + 4 = 0$
$3x(x - 4) - 1(x - 4) = 0$
$(x - 4)(3x - 1) = 0$
$x = 4, \frac{1}{3}$
So, A is false but R is true.

20. **(c)** A is true but R is false.
Explanation: We have,
LCM(a, b) × HCF(a, b) = a × b
LCM × 5 = 150
LCM = $\frac{150}{5}$ = 30
LCM = 30

Section B

21. x + y = 14 ...(1)
x − y = 4 ... (2)
If x - y = 4
x = 4 + y .... (3)
Put (3) in (1) we get
4 + y + y = 14
⇒ 4 + 2y = 14
⇒ 2y = 10
⇒ y = 5
Putting value of y in equation (3), we get
x = 4 + y
⇒ x = 4 + 5 = 9
Therefore, x = 9 and y = 5

22. Let the polynomial be $ax^2 + bx + c$,
and its zeroes be α and β.
Then, $\alpha + \beta = 0 = -\frac{b}{a}$ and $\alpha\beta = \sqrt{5} = \frac{c}{a}$
If a = 1, then b = 0 and $c = \sqrt{5}$.
So, one quadratic polynomial which fits the given conditions is $x^2 + \sqrt{5}$.

23. According to question we are given that a group consists 12 persons, in which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind.
∴ Total number of outcomes = 12
i. Given , number of extremely patient persons = 3
∴Number of favourable outcomes = 3
∴ P(extremely patient) = $\frac{3}{12} = \frac{1}{4}$
ii. Given , number of extremely honest persons = 6
and number of extremely kind persons = 12 - 6 - 3 = 3
∴ Number of favourable outcomes = Number of extremely kind persons + Number of extremely honest persons
= 6 + 3 = 9

$\therefore$ P(extremely kind or honest) $= \frac{9}{12} = \frac{3}{4}$

Value preferred is Honesty

24. Length of the tangents drawn from an external point to a circle are equal.

$\therefore CA = BA = 2.8cm$ ...(i)

$AB = AP = 2.8cm$ ...(ii)

From equation (i) and (ii) :

$CA = AB = 2.8cm$

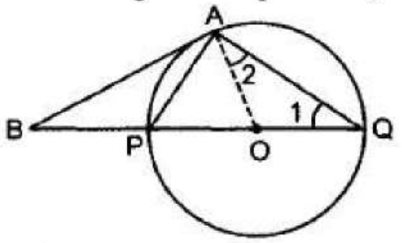
$CB = CA = AB$

$\therefore BC = 2.8 + 2.8$

BC = 5.6 cm

OR

According to the question,



$PAQ = 90^o$ [Angle in semicircle]

$\therefore \quad \angle APQ + \angle 1 = 90^\circ$ [Sum of acute angels of right angle $\triangle$]

$\Rightarrow \angle APQ + \angle 2 = 90^\circ$ [OA = OQ $\therefore \angle 1 = \angle 2$]

$\Rightarrow \angle APQ + (\angle BAQ - \angle BAO) = 90^\circ$

$\Rightarrow \angle APQ + (105^\circ - 90^\circ) = 90^\circ$ [$\because$ OA $\perp$ **AB**]

$\Rightarrow \angle APQ = 90^\circ - 15^\circ = 75^\circ$

25. Distance between the given points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 5\sin 60^\circ)^2 + (5\sin 30^\circ - 0)^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2} \left\{\because \sin 60^\circ = \frac{\sqrt{3}}{2}, \sin 30^\circ = \frac{1}{2}\right\}$

$= \sqrt{\frac{25\times3}{4} + \frac{25\times1}{4}} = \sqrt{\frac{75}{4} + \frac{25}{4}}$

$= \sqrt{\frac{100}{4}} = \sqrt{25} = 5$ units

OR

We have to find a point on the y-axis which is equidistant from the points A(6, 5) and B (-4, 3).

We know that a point on y-axis is of the form (0, y). So, let the required point be P (0, y). Then,

PA = PB

$\Rightarrow \sqrt{(0-6)^2 + (y-5)^2} = \sqrt{(0+4)^2 + (y-3)^2}$

$\Rightarrow 36 + (y - 5)^2 = 16 + (y - 3)^2$

$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

So, the required point is (0, 9).

Section C

26. We have,

$5\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5}$

Now, L.H.S $= \frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{\frac{5\sin\theta - 3\cos\theta}{\cos\theta}}{\frac{5\sin\theta + 2\cos\theta}{\cos\theta}}$ [Dividing Numbers and Denominator by $\cos\theta$]

$= \frac{\frac{5\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{5\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}} = \frac{5\tan\theta - 3}{5\tan\theta + 2} = \frac{5\times\frac{4}{5} - 3}{5\times\frac{4}{5} + 2}$ $[\because \tan\theta = \frac{4}{5}]$

$= \frac{4-3}{4+2} = \frac{1}{6}$ = R.H.S

Hence proved.

27. The given system of equations is:

$x + 2y = -1$ ...(i)

$2x - 3y = 12$ ...(ii)

From equation (i), we get

$x = -1 - 2y$

Substituting x = -1 - 2y in equation (ii), we get

$2(-1 - 2y) - 3y = 12$

$\Rightarrow -2 - 4y - 3y = 12$

$\Rightarrow -7y = 14$

$\Rightarrow y = -2$

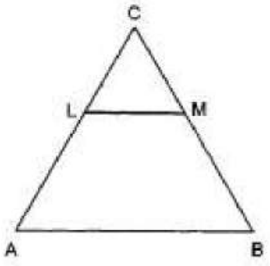
Putting y = -2 in x = -1 - 2y, we get

x = -1 - 2 × (-2) = 3

Hence, the solution of the given system of equations is x=3, y=-2.

28. We have, AL = x - 3, AC = 2x, BM = x - 2 and BC = 2x + 3, and we need to find the value of x.

In ΔABC, we have



$LM \| AB$

$\therefore \quad \frac{AL}{LC} = \frac{BM}{MC}$ [By Thaley's Theorem]

$\Rightarrow \quad \frac{AL}{AC-AL} = \frac{BM}{BC-BM}$

$\Rightarrow \quad \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$

$\Rightarrow \quad \frac{x-3}{x+3} = \frac{x-2}{x+5}$

$\Rightarrow$ (x - 3) (x + 5) = (x - 2) (x + 3)

$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$

$\Rightarrow$ x = 9

29. We have to prove that $\sqrt{2}$ is an irrational number.

Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \frac{p}{q}$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

or, $2 = \frac{p^2}{q^2}$

or, $p^2 = 2q^2$

$\therefore$ p^2 is divisible by 2.

p is divisible by 2........(i)

Let p = 2r for some integer r

or, $p^2 = 4r^2$

$2q^2 = 4r^2$ [$\because p^2 = 2q^2$]

or, $q^2 = 2r^2$

or, q^2 is divisible by 2.

$\therefore$q is divisible by 2..........(ii)

From (i) and (ii)

p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore \sqrt{2}$ is irrational number.

OR

Given, $p = a^2b^3$

and $q = a^3b$

HCF(p,q) = a^2b

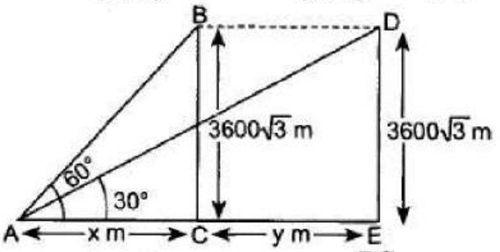
LCM(p, q) = a^3b^3

$$pq = a^2b^3 \times a^3b = a^5b^4 \text{ ------------ } (1)$$

$$LCM(p,q) \times HCF(p,q) = a^3b^3 \times a^2b = a^5b^4 \text{ ----}(2)$$

From equation (1) *and* (2) *We get*

$$LCM(p,q) \times HCF(p,q) = pq$$

30. 

In rt. $\triangle$ACB, $\tan 60^\circ = \frac{BC}{AC}$

$\sqrt{3} = \frac{3600\sqrt{3}}{x}$

$x = 3600\ m$

Now, In right AED,

$\tan 30^\circ = \frac{DE}{AE}$

$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{3600+y}$

$3600 + y = 10800$

$y = 7200m$

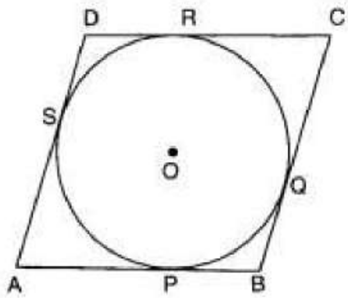
$BD = CE$

$\therefore$ Distance covered in 30 seconds = 7200 ,

So, Speed = $\frac{7200}{30} = 240\text{m/s}$

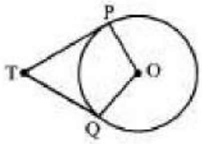
$= 240 \times \frac{18}{5}$

$= 864\ km/hr.$

31. 

Let ABCD be a parallelogram such that its sides touch a circle with centre O.
We know that the tangents to a circle from an exterior point are equal in length.
Therefore, AP = AS [From A] ...(i)
BP = BQ [FromB] ...(ii)
CR = CQ [From C] ...(iii)
and , DR = DS [From D] ...(iv)
Adding (i), (ii), (iii) and (iv), we get,
AP + BP + CR + DR = AS + BQ +CQ + DS
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)
$\Rightarrow$ AB + CD = AD + BC
$\Rightarrow$ 2 AB = 2 BC
$\Rightarrow$ AB = BC
Therefore, AB = BC = CD = AD
Thus, ABCD is a rhombus.

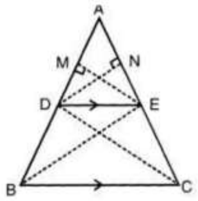
OR



Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact.
PT and QT are tangents to the circle with centre O.
$\therefore$ OP $\perp$ PT and OQ $\perp$ QT
i.e., $\angle POQ = 110°$, $\angle OPT = 90°$ $\angle OQT = 90°$
In quad OPTQ we have,
$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360°$
$\angle PTQ = 360° - (110° + 90° + 90°)$
$\angle PTQ = 360° - 290°$
$\angle PTQ = 70°$

Section D

32. Given: ABC is a triangle in which DE || BC.
To prove: $\frac{AD}{BD} = \frac{AE}{CE}$
Construction: Draw $DN \perp AE$ and $EM \perp AD$., Join BE and CD.
Proof :



In $\triangle ADE$,

Area of $\Delta ADE = \frac{1}{2} \times AE \times DN$ ...(i)
In ΔDEC,
Area of $\Delta DCE = \frac{1}{2} \times CE \times DN$ ...(ii)
Dividing equation (l) by equation (ii),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{AE}{CE}$ ...(iii)
Similarly, In ΔADE,
Area of $\Delta ADE = \frac{1}{2} \times AD \times EM$ ...(iv)
In ΔDEB,
Area of $\Delta DEB = \frac{1}{2} \times EM \times BD$ ...(v)
Dividing equation (iv) by equation (v),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AD}{BD}$ ...(vi)
ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.
$\therefore$ Area (ΔDEB) = Area (ΔDEC)
From equation (iii),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AE}{CE}$. ......(vii)
From equation (vi) and equation (vii),
$\frac{AE}{CE} = \frac{AD}{BD}$
$\therefore$ If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio.

33. Total time taken by minute hand from 2 p.m. to 3 p.m. is 60 min.
According to question,
$t + \left(\frac{t^2}{4} - 3\right) = 60$
$\Rightarrow 4t + t^2 - 12 = 240$
$\Rightarrow t^2 + 4t - 252 = 0$
$\Rightarrow t^2 + 18t - 14t - 252 = 0$
$\Rightarrow t(t + 18) - 14(t + 18) = 0$
$\Rightarrow (t + 18)(t - 14) = 0$
$\Rightarrow t + 18 = 0$ or $t - 14 = 0$
$\Rightarrow t = -18$ or $t = 14$ min.
As time can't be negative.
Therefore, t = 14 min.

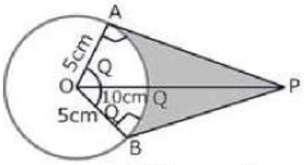
OR

Let cost of production of each article be Rs x
We are given total cost of production on that particular day = Rs 90
Therefore, total number of articles produced that day = 90/x
According to the given conditions,
$x = 2\left(\frac{90}{x}\right) + 3$
$\Rightarrow x = \frac{180}{x} + 3$
$\Rightarrow x = \frac{180 + 3x}{x}$
$\Rightarrow x^2 = 180 + 3x$

$\Rightarrow x^2 - 3x - 180 = 0$
$\Rightarrow x^2 - 15x + 12x - 180 = 0$
$\Rightarrow$ x (x − 15) + 12 (x − 15) = 0
$\Rightarrow$ (x − 15) (x + 12) = 0 $\Rightarrow x = 15, -12$
Cost cannot be in negative, therefore, we discard $x = -12$
Therefore, x = *Rs* 15 which is the cost of production of each article.
Number of articles produced on that particular day = $\frac{90}{15} = 6$

34. Let the radii of the circles are r_1 cm and r_2 cm
$\therefore r_1 + r_2 = 14$ ...(i)
And, sum of their areas = $\pi r_1^2 + \pi r_2^2$
$130\pi = \pi\left(r_1^2 + r_2^2\right)$
or, $130\pi = \pi\left(r_1^2 + r_2^2\right)$
$\therefore \quad r_1^2 + r_2^2 = 130$ ...(ii)
$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$
or, $(14)^2 = 130 + 2r_1r_2$
or, $2r_1r_2 = 196 - 130$
0r, $2r_1r_2 = 66$
$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$
$(r_1 - r_2)^2 = 130 - 66$
$(r_1 - r_2)^2 = 64$
or, $r_1 - r_2 = 8$ ...(iii)
From (i) and (iii), $2r_1 = 22$
or, $r_1 = 11$ cm
$r_2 = 14 - 11$
$r_2 = 3\ cm$.

OR



$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$
$\Rightarrow \theta = 60^o$
$\Rightarrow \angle AOB = 2 \times \theta = 120^o$
$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times 5}{360} cm = \frac{10\pi}{3} cm \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$
Length of the belt that is in contact with the rim of the pulley
= Circumference of the rim - length of arc AB
= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm
= $\frac{20\pi}{3}$cm
Now, the area of sector OAQB = $\frac{120\times\pi\times 5\times 5}{360} cm^2 = \frac{25\pi}{3} cm^2 \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$
Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2
$\left[\because AP = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm\right]$
Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

35. Calculation of median:

Class interval	Frequency(f_i)	Cumulative frequency

5 - 10	5	5
10- 15	6	11
15 - 20	15	26
20 - 25	10	36
25 - 30	5	41
30 - 35	4	45
35 - 40	2	47
40 - 45	2	49

Now,N = 49 $\Rightarrow \frac{N}{2} = 24.5$.

Thus, the median class is 15 - 20.

$\therefore$ l = 15, h = 5, f = 15, c.f. = 11

Median, $M = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$

$= 15 + \left(5 \times \frac{(24.5-11)}{15}\right)$

$= 15 + \left(5 \times \frac{13.5}{15}\right)$

$= 15 + 4.5 = 19.5$

Hence, the median of frequency distribution is 19.5

Section E

36. **Read the text carefully and answer the questions:**

Jaspal Singh is an auto driver. His autorickshaw was too old and he had to spend a lot of money on repair and maintenance every now and then. One day he got to know about the EV scheme of the Government of India where he can not only get a good exchange bonus but also avail heavy discounts on the purchase of an electric vehicle. So, he took a loan of ₹1,18,000 from a reputed bank and purchased a new autorickshaw.

Jaspal Singh repays his total loan of 118000 rupees by paying every month starting with the first instalment of 1000 rupees.

(i) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Now, amount paid in the 30th installment,
$a_{30} = 1000 + (30 - 1)100 = 3900$ $\{a_n = a + (n - 1)d\}$

(ii) Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 30 instalments,

$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100] = 73500$

Hence, remaining amount of loan that he has to pay = 118000 - 73500 = 44500 Rupees

OR

Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month

therefore, installment amount in second month = ₹ 1100, third month = ₹1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment

Then a = 1000, d = 200 and n = 40

$a_{40} = a + (n - 1)d$

$\Rightarrow a_{40} = 1000 + (40 - 1)200$

$\Rightarrow a_{40} = 880$

(iii)Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month

therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Amount paid in 100 instalments

$S_n = \frac{n}{2}[2a + (n - 1)d]$

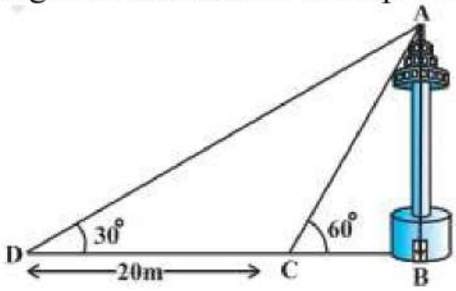
$S_n = \frac{100}{2}[2 \times 1000 + (100 - 1)100]$

$\Rightarrow S_n = 100000 + 9900$

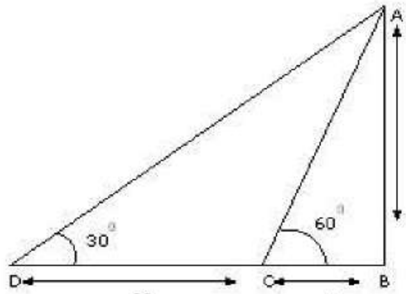
$\Rightarrow 109900$

37. **Read the text carefully and answer the questions:**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30°.



(i)



Let 'h' (AB) be the height of tower and x be the width of the river.

In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ...(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^\circ$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii),

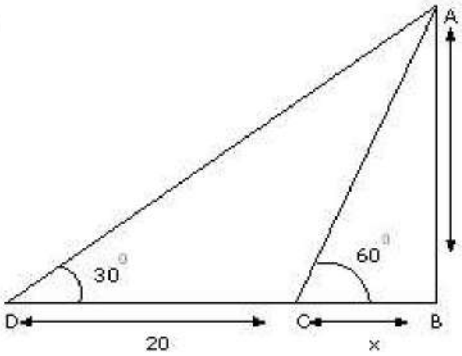
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow$ 3x = x + 20

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10 m

(ii)



Let 'h' (AB) be the height of tower and x be the width of the river.

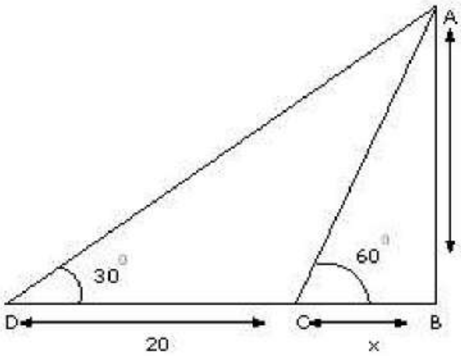
In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ...(i)

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}$m

OR



In $\triangle$ABC

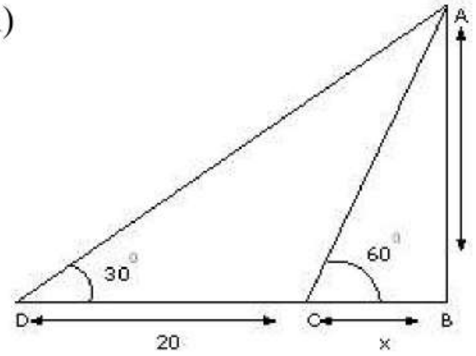
sin $60^0 = \frac{AB}{AC}$

$\Rightarrow$ AC = $\frac{AB}{\sin 30^0}$

$\Rightarrow$ AC = $\frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AC = 20 m

(iii)



In $\triangle ABD$

$\sin 30^{o} = \frac{AB}{AD}$

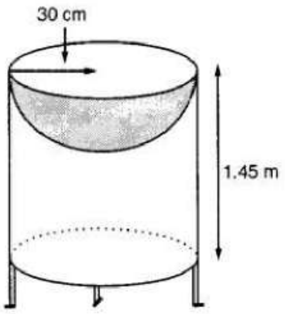
$\Rightarrow AD = \frac{AB}{\sin 30^{\circ}}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow AD = 20\sqrt{3}$ m

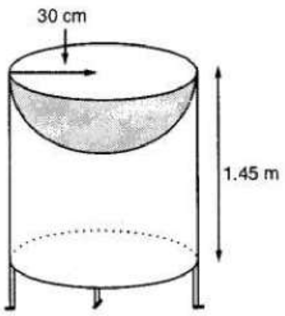
38. **Read the text carefully and answer the questions:**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

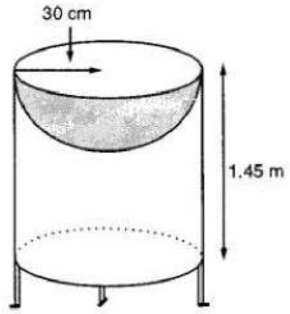


Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



Let S be the total surface area of the birdbath.

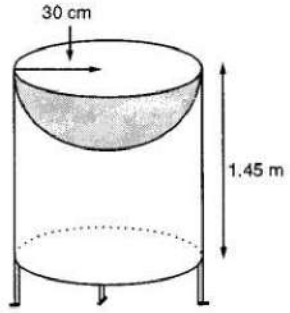
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) \quad = 33000 \text{ cm}^2 = 3.3 \text{ m}^2$

(iii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



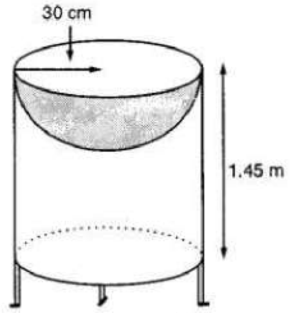
Total Cost of material = Total surface area × cost per sq m^2

= 3.3 × 40 = ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



$r = 35 \text{ cm} = \frac{35}{100} \text{m}$

We know that S.A = 3.3 m^2

S = 2πr(r + h)

$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$

$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$

$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$

$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20} \quad = 1.15 \text{ m}$